

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Number

Finding the HCF and LCM using prime factor decomposition

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

- (a) Identifies at least 1 pair of factors

2 (×) 63, 3 (×) 42, 6 (×) 21, 7 (×) 18, 9 (×) 14

Accept eg 3,6,7

Do not accept 1 (×) 126

M1

$2 \times 3 \times 3 \times 7$ oe must see multiplication signs

SC1 for 2 (×) 3 (×) 7

A1

- (b) Identifies at least 1 pair of factors

2 (×) 36, 3 (×) 24, 4 (×) 18, 6 (×) 12, 8 (×) 9

or 2 (×) 2 (×) (2) (×) 3 (×) 3

Do not accept 1 (×) 72

M1

18

SC1 for 6 or 9 or $2 \times 3 \times 3$

A1 ft**[4]****Q2.**

- Any correct product of 36 using a prime factor

2 and 18

2 and 2 and 9

3 and 12

3 and 3 and 4

2 and 3 and 6

May be on a factor tree or repeated division

M1

2 and 2 and 3 and 3 oe May be on a factor tree or repeated division

A1

$2^2 \times 3^2$ or $3^2 \times 2^2$

A1**Additional Guidance**

Allow any number of 1s included as factors for up to M1A1 only

$1 \times 2^2 \times 3^2$

M1A1A0

$2^2 \cdot 3^2$

M1A1A1

$2 + 2 + 3 + 3$

M1A1A0

$2^2 + 3^2$

M1A1A0

$2^2 3^2$ or $2^2, 3^2$

M1A1A0

$2 \times 2 \times 3 \times 3$ and $2^2 \times 3^2$ on answer line

M1A1A0

but $2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ on answer line

M1A1A1

$2^2 \times 3^2 = 6^2$

M1A1A0

6×6 with no prime factorisation

M0A0A0**[3]**

Q3.

200 written as a product of factors where at least one factor is prime

Eg 2 and 100 or 2×10^2 or $200 \div 5 = 40$

may be on a factor tree or repeated division

allow one strand to be incorrect if a previous value completes the product

eg 10×20 followed by

$5 \times 2 \times 5 \times 6$ implies $5 \times 2 \times 20$ for M1

M1

2 and 2 and 2 and 5 and 5 *may be on a factor tree or repeated division*

A1

$2^3 \times 5^2$ or $5^2 \times 2^3$

A1

Additional Guidance

Allow any number of 1s included as factors up to M1A1 only

M1 may be awarded for correct work with no or incorrect answer, even if this is seen among multiple attempts

$1 \times 2^3 \times 5^2$

M1A1A0

$2^3.5^2$ or $2^3.5^2$ or 2^35^2 or $2^3,5^2$

M1A1A1

$2 + 2 + 2 + 5 + 5$

M1A1A0

$2^3 + 5^2$

M1A1A0

$2 \times 2 \times 2 \times 5 \times 5$ and $2^3 \times 5^2$ on answer line

M1A1A0

but $2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$ on answer line

M1M1A1

$2^3 \times 5^2 = 10^5$

M1A1A0

$2^3 \times 5^2 = 200$

M1A1A1

8×25 with no prime factorisation

M0A0A0

[3]

Q4.

(a) 18 (×) 2 or 12 (×) 3 or 9 (×) 2 (×) 2

Allow on prime factor tree or repeated division

or 3 (×) 3 (×) 4 or 2 (×) 3 (×) 6

Condone 18 (×) 2 (×) 1 etc. for this mark

M1

2 (×) 2 (×) 3 (×) 3

Allow on prime factor tree or repeated division

A1

$2^2 \times 3^2$

ft any product of prime numbers in index form if M1 awarded

A1ft

(b) 9

B1 answer 3 or 3^2 or 3×3

or $81 = 3^4$ or 3 (×) 3 (×) 3 (×) 3

B2

[5]

Q5.

(a) 6

B1 for answer of 2 or 3 or 2×3

B1 for $24 = \{2, 2, 2, 3\}$ or $2 \times 2 \times 2 \times 3$

or $42 = \{2, 3, 7\}$ or $2 \times 3 \times 7$

or

one pair of factors of 24

(not 1×24) eg 2×12 , 3 and 8, $24 \div 4 = 6$ (oe)

and *one pair of factors of 42 (not 1×42)*

eg 2×21 , 3 / 14, (6, 7) (oe)

or $(24 =) \{1, 2, 3, 4, 6, 8, 12, 24\}$

or $(42 =) \{1, 2, 3, 6, 7, 14, 21, 42\}$

B2

(b) 48 as a correct product

(except 1×48)

eg

2×24 or 3×16 or 6×8 or

$2 \times 3 \times 8$ or (2, 24) or (1, 2, 3, 8) etc

oe eg $48 \div 2 = 24$ or branches on a prime factor tree showing at least one product or factor ladder showing a correct division

Ignore incorrect products if at least one correct product seen

M1

$2 \times 2 \times 2 \times 2 \times 3$ or $2^4 \times 3$ ⁽¹⁾

or $2^3 \times 2 \times 3$ ⁽¹⁾ or $2^2 \times 2^2 \times 3$ ⁽¹⁾

A1

[4]

Q6.

2 (×) 140 or 5 (×) 56 or 7 (×) 40

oe Correct product with at least one prime factor

M1

$2 \times 2 \times 2 \times 5 \times 7$

oe

A1

[2]

Q7.

2 (×) 150 or 3 (×) 100 or 5 (×) 60

oe Correct product with at least one prime factor

M1

$2 \times 2 \times 3 \times 5 \times 5$

oe

A1

[2]

Q8.

2 (×) 70
or 5 (×) 28
or 7 (×) 20

May be on a diagram

M1

$2 \times 2 \times 5 \times 7$

Any order

A1

$2^2 \times 5 \times 7$

Any order

A1

[3]

Q9.

3 (×) 75 or 5 (×) 45
or 3 (×) 3 (×) 25 or 5 (×) 5 (×) 9
or 3, 3, 5, 5

May be seen on a factor tree

M1

$3 \times 3 \times 5 \times 5$ or $3^2 \times 5^2$

In any order

oe

ie $3 \times 3 \times 5^2$

$3^2 \times 5 \times 5$

A1

[2]

Q10.

(a) 2×25 or 5×10

*oe eg $50 \div 2 = 25$ **or** branches on a prime factor tree **or** any indication eg (2, 25) of a 'product' that equals 50 **or** 2, 5, 5 **or** 2, 5 and 5 shown as the last numbers of a prime factor tree (allow 1s)*

M1

$2 \times 5 \times 5$

$2^{(1)} \times 5^2$

A1

(b) List of multiples of 40 and 50 to at least 80, 120 and 100, 150

Venn diagram (ft their prime factors for 50 in (a))

M1

$2^3 \times 5^2$ or 200

oe SC1 any multiple of 200

A1

[4]

Q11.

Any correct value

11, 23, 37, 53, 71, 91, 113, 137, 163

M1

Selects 91 as the only incorrect value with no errors in values given

oe

eg stops at 91

A1

91 and 13 (is a factor)

or

91 and 7 (is a factor)

or

91 and 13×7

oe

eg $91 \div 7 = 13$

A1

Additional Guidance

Ignore incorrect evaluations for first mark

Ignore all values for n greater than 9

Do not allow 11 within a list of prime numbers eg 2, 3, 5, 7, 11...

Error in list eg 12, 23, 37, 53, 71, 91, 113, 137, 163 with 12 and 91 selected as not prime (not valid as incorrect)

M1A0A0

Error in list eg 12, 23, 37, 53, 71, 91, 113, 137, 163 with only 91 selected as not prime (not valid as incorrect conclusion from their list)

M1A0A0

$9^2 + 9 + 1 = 91$ is incorrect working

M0A0A0

[3]

Q12.

- (a)
- 2×100
- or
- 5×40

or conditional on one prime factor in a correct product equal to 200 or one prime factor shown in a correct section on a factor tree starting from 200

Any order

allow on prime factor tree or repeated division using 2 or 5 correctly

condone $100 \times 2 \times 1$ etc for this mark

M1

$2 \times 2 \times 2 \times 5 \times 5$

Any order

allow on prime factor tree or repeated division

A1

$2^3 \times 5^2$

Strand (i) correct index notation

Any order

ft correct product of prime numbers in index form from their working

Q1ft**Additional Guidance**

$2^3 + 5^2$

M1A1Q0

$(200 =) 2 \times 2 \times 5 \times 5$ and $2^2 \times 5^2$ is minimum Q1ft

$200 \div 2 = 100$

M1

$2 \times 10 \times 10$ as a product or shown on a correct section of factor tree

M1

$20 \times 5 \times 2$ as a product or shown on a correct section of factor tree

M1

$20 \times 5 \times 4$ as a product or shown on a correct section of factor tree

M0

- (b) 4 and 60
- and**
- 12 and 20

B1 one correct

or one correct and one incorrect

or two correct and one incorrect

Any indication

B2**[5]**

Q13.

- (a) 2×66 or 3×44 or $2 \times 6 \times 11$
 or $3 \times 4 \times 11$ or 12×11
 or $2 \times 2 \times 33$ or $2 \times 3 \times 22$

Any order

Allow on prime factor tree or repeated division.

Condone 2×66 66×1 etc

M1

$$2 \times 2 \times 3 \times 11$$

$$\text{or } 2^2 \times 3 \times 11$$

Any order

A1

Additional Guidance

2, 2, 3, 11

M1A0

- (b) **Alternative method 1**

$$2 \times 5 \times 11 = 110$$

M1

$$22$$

SC1 11

A1

Alternative method 2

List of factors of 110 **and** 132 up to 22 with 2 errors or omissions

(1), 2, 5, 10, 11, 22 (55, 110)

and

(1), 2, 3, 4, 6, 11, 12, 22

(33, 44, 66, 132)

M1

$$22$$

SC1 11

A1

Additional Guidance

(1, 55, 110) and (1, 33, 44, 66, 132) are not omissions

[4]

Q14.

$$28 \times 2 \text{ or } 8 \times 7 \text{ or } 14 \times 2 \times 2$$

$$\text{or } 2 \times 4 \times 7$$

$$\text{or } 2, 2, 2, 7$$

allow on prime factor tree or repeated division

ignore incorrect products if at least one correct product seen

M1

$$2 \times 2 \times 2 \times 7 \text{ or } 2^3 \times 7$$

A1

Additional Guidance

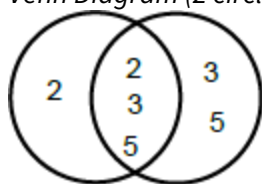
Ignore any $\times 1$ for M1 but not A1

[2]

Q15.

- (a) 30 or $2 \times 3 \times 5$

Venn Diagram (2 circles) with correct values



or Lists of multiples of 60 or 450

eg 120, 180, 240, ...

450, 900,

B1

- (b) 60 and 450 seen

Sc1. No working or irrelevant working and answer of 1800

B2

[3]

Q16.

- (a) Correct product using at least one prime factor

For example

$2 (\times) 126$ or $3 (\times) 84$ or

$7 (\times) 36$ or $2 (\times) 2 (\times) 63$ or

$2 (\times) 3 (\times) 42$

May be implied

eg in a factor tree or repeated division

M1

$$2 \times 2 \times 3 \times 3 \times 7 \text{ or}$$

$$2^2 \times 3^2 \times 7$$

A1

- (b) 84

B1

[3]

Q17.

- (a) Pair of values of form $2x$ and kx
where x is an integer > 5
and k is an odd integer > 2
eg 12 and 18 or 100 and 250

B1

- (b) Ticks No with valid reason including that one number could be 2 and that multiplying by an even number (or 2) gives an even answer

*eg1 Ticks No and a could be 2
and the others will be odd*

and even $\times$ odd $\times$ odd = even

*eg2 Ticks No (and b could be 2)
and $27 \times 4 \times 5 = 540$*

eg3 Ticks No and $125 \times 9 \times 2 = 2250$

*eg4 Ticks No and a, b or c could be 2 and
multiplying by an even gives an even*

B1 No with partial reason

eg1 Ticks No and a could be even

eg2 Ticks No and $c = 2$

*eg3 Ticks No and odd $\times$ odd $\times$ even is
even*

*eg4 Ticks No and multiplying by an even
gives an even*

B2

Additional Guidance

If a box is not ticked, No can be implied by the reason for B2 and B1

eg1 a could be 2 and the others will be odd and even $\times$ odd $\times$ odd = even

B2

eg2 b could be 2 and $27 \times 4 \times 5 = 540$ which is even

B2

eg3 odd $\times$ odd $\times$ even is even

B1

Allow use of numbers that are not prime for B1

eg1 Ticks No and b could be 2 and $1 \times 4 \times 5 = 20$

B1

eg2 Ticks No and $4^3 \times 3^2 \times 2 = 1152$

B1

[3]

Q18.

$$3^8 \text{ or } 3^9 \text{ or } y^6$$

$$\text{or } 2 \times 3^4 \times y^3 \times 2 \times 3^4 \times y^3$$

$$\text{or } 3 \times 2 \times 3^4 \times y^3 \times 2 \times 3^4 \times y^3$$

$$78\,732 \text{ or } 19\,683$$

M1

$$2^2 \times 3^8 \times y^6$$

$$\text{or } 3 \times 2^2 \times 3^8 \times y^6$$

$$\text{or } 2^2 \text{ and } 3^9 \text{ and } y^6$$

$$\text{or } 2^a \times 3^b \times y^c$$

with two powers correct

$$2^2 \times 19\,683y^6$$

$$78\,732y^6$$

M1dep

$$2^2 \times 3^9 \times y^6$$

Must be in index form

Do not ignore fw

A1

Additional Guidance

$$2^2 \times 3^8 \times y^6$$

M1 M1 A0

$$2^2 + 3^9 \times y^6$$

M1 M1 A0

$$2^2 + 3^8 + y^6$$

M1 M0 A0

[3]

Q19.

$$80$$

$$(200 =) 2^3 \times 5^2$$

$$\text{or } 2^4 \times 5 \text{ oe}$$

$$\text{or } 16 \times 5$$

$$\text{or } 200 \times 2 \div 5$$

$$\text{B1 } a = 2 \text{ and } b = 5$$

$$\text{or } 2, 2, 2, 5, 5 \text{ seen on a factor tree}$$

$$\text{or } 25$$

$$\text{or } 8$$

B3

Additional Guidance

For B1, 25 or 8 must be chosen from any lists of square or cube numbers

$$2 \times 2 \times 2 \times 5 \times 5$$

B2

$$5^3 \times 2^2$$

B0

[3]

Q20.

$$e = c^2d$$

B1

[1]

Q21.

(a) $2^3 \times 3 \times a^2$ or $24a^2$ (= 4056)

or

$$a = \frac{4056}{2^3 \times 3}$$

or $a^2 = 169$

or

$$\sqrt{169}$$

oe eg $8 \times 3 \times a^2$

M1

13

A1

Additional Guidance

Condone $a^2 \times 24$ for M1

Fully correct prime factor decomposition with values 2, 2, 2, 3, 13, 13 shown without 13 chosen as the final answer

M1A0

Embedded answer $2^3 \times 3 \times 13^2$

M1A0

± 13 or -13

M1A0

$4056 \div 2^3 \times 3$ unless recovered to 169

M0A0

(b) $2^4 \times 3^2 \times a^3$ or $144a^3$

or

$$2^4 \times 3^2 \times (\text{their } 13)^3$$

or

$$13 \times 4056 \times 2 \times 3$$

or

$$52728 \times 6$$

or

$$24336 \times 13$$

oe

$$\text{eg } 144 \times (\text{their } 13)^3$$

$$16 \times 9 \times 2197$$

M1

316 368

ft their 13, which must be an integer > 13

A1ft

Additional Guidance

eg 14 on answer line in part (a) can follow through to $144 \times 14^3 = 395136$

M1A1ft

[4]

Q22.

- (a) 4 and 5
or 4 and 7
or 4 and 9

B1

- (b) Ticks No with valid reason

eg Ticks No, the smallest value of N is $2 \times 3 \times 5 = 30$.

or

Ticks No, you can only get $N = 6$ from $1 \times 2 \times 3$ and 1 is not a prime number.

B1 No with partial reason

eg Ticks No and it can't be 1.

B2

Additional Guidance

If a box is not ticked, No can be implied by the reason for B2 and B1

eg1 the smallest value of N is $2 \times 3 \times 5 = 30$.

B2

eg2 you can only get $N = 6$ from $1 \times 2 \times 3$ and 1 is not a prime number.

B2

[3]